Physics 211C: Solid State Physics

Instructor: Prof. Tarun Grover

Topic 1: Fermi Liquid Theory

Basic idea (from notes): Adiabatic continuity relates interacting problem to non-interacting one.

Steps:- ⓐ • Start from non-interacting $\sum_k \epsilon_k c_k^+ c_k$

ⓑ • Slowly turn on interactions ("slow" quantified later)

ⓒ • If the ⓑ can be done at all, the new ground state is

$$|\Psi_t\rangle = T e^{-i\int_{-\infty}^{0} H_I(t)\,dt} \underbrace{|\Psi_{-\infty}\rangle}_{|FS\rangle}$$

where

$$\mathcal{H} = \underset{\substack{\uparrow \\ \text{bare}}}{\mathcal{H}_0} + H_I(t=0) \equiv H_0 + H_I$$

Landau FLT states that if the above procedure can be carried out without encountering any phase transitions, then the low lying eigenstates $\left(\Delta E \sim \frac{1}{L^{\#}}\right)$ of $\mathcal{H}$ are 1-1 correspondence with eigenstates of H_0.

Precisely, if $|\Psi_{t=-\infty}\rangle = \prod_{k<k_F} c_k^+ |0\rangle$

$$\Downarrow$$

$$|\Psi_{t=0}\rangle = \prod_{k<k_F} a_k^+ |0\rangle$$

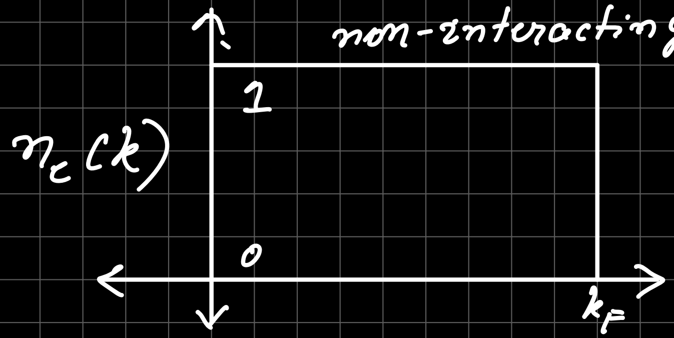
where $a_k^+ = U c_k^+ U^+$ is the QP operator

FLT assumes that a_k^+ & c_k^+ are related as

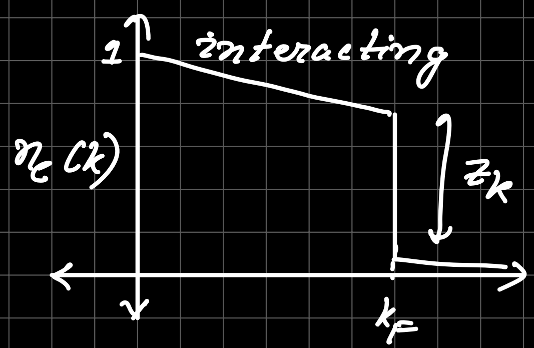
$$c_{k\sigma}^+ = \sqrt{Z}\, a_{k\sigma}^+ + A(\{k\})\, a_{k\sigma}^+ a_{k_1\sigma'}^+ a_{k_1\sigma'} + \cdots$$

where crucially $\sqrt{Z_k} \neq 0$ (can be taken to be almost the def^n of FLT)

Correspondingly, $n_k = \langle c_{k\sigma}^+ c_{k\sigma} \rangle$ has a discontinuity of Z_k at FS



$\Longrightarrow$



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We introduce Green's functions to analyse Fermi liquid.

Single Particle Green's function in a Fermi liquid

$$G(t,k) = -i \langle g.s. | T\, c_k(t)\, c_k^+(0) | g.s. \rangle$$

$$= -i\, \Theta(t) \langle gs | c_k(t) c_k^+(0) | gs \rangle$$

$$+ i\, \Theta(-t) \langle gs | c_k^+(0)\, c_k(t) | gs \rangle$$

$$c_k(t) = e^{-it\tilde{\xi}_k} c_k(0) \qquad \tilde{\xi}_k = \xi_k = E_k - \mu$$

$$\langle gs | c_k c_k^+ | gs \rangle = 1 - n_F \quad \text{(for free fermions)}$$

$$\therefore G(t,k) = \underbrace{-i\, \Theta(t)\, e^{-i\xi_k t} (1-n_F)}_{\Downarrow} + i\, \Theta(-t)\, n_F\, e^{-i\xi_k t}$$

$$n_F = \Theta(\xi_k)$$

$$\Rightarrow G(t,k) = -i\, \Theta(t) e^{-i\xi_k (t)}\, \Theta(\xi_k) + i\, \Theta(-t)\, \Theta(-\xi_k)\, e^{-i\xi_k t}$$

$$\Big\downarrow \text{ f.t.}$$

$$G(\omega, k) = \frac{1}{\omega - \tilde{\xi}_k + i \cdot \epsilon \cdot \underbrace{\mathrm{sign}(\omega)}_{\text{sign of real part of } \omega}}$$

$\therefore \quad G^{-1}_{free}(k,\omega) = \omega - \tilde{\xi}_k + i\eta\, \mathrm{sign}(\omega)$

* All this was for non-interacting system. For interacting system,

$$G^{-1}_{int}(k,\omega) = \omega - \tilde{\xi}_k - \underbrace{\Sigma(k,\omega)}_{\text{self energy}}$$

e.g:- ① $\Sigma(k,\omega) = \Delta\tilde{\xi}_k \Rightarrow$ new FS

$\tilde{\xi}_k + \Delta\tilde{\xi}_k = 0$

② $$\frac{1}{\omega - \tilde{\xi}_k + \frac{i}{\tau}} \xrightarrow{f.T.} \underbrace{e^{-t/\tau}}_{\text{Quasi-particle lifetime.}}$$

τ = life-time of q.p.

$\Sigma(k,\omega)$ is a model dependent quantity. It has to be calculated in a perturbative way.

(TG:- Landau theory $\Leftrightarrow$

hubbard model, large N, hubbard model in momentum space

"1d" integrable models

Bethe Ansatz solvable $\Rightarrow$

* integrability isn't well defined. $\Rightarrow$ still modern)

Expand G^{-1} near FS

$\Sigma = \mathrm{Re}\,\Sigma + i\,\mathrm{Im}\,\Sigma$

$$G^{-1} \simeq \omega - \omega\left(\frac{\partial \mathrm{Re}\,\Sigma}{\partial \omega}\right)_{\omega=0}$$

$$- (k-k_F)\, \partial_k \left(\xi_k + Re\,(\Sigma(k,0)\right)\Big|_{k=k_F}$$

$$- i\, Im\left(\Sigma(k,\omega)\right)$$

$$Z^{-1} = 1 - \frac{\partial}{\partial \omega}\left(Re\,\Sigma(k_F,\omega)\right)$$

$$\Downarrow$$

$$G \sim \frac{1}{\omega Z^{-1} + 1} = \frac{Z}{\omega - (\quad)}$$

$$\therefore\ G = \frac{Z}{\omega - \xi'_{renom} - \frac{i}{\tau(k,\omega)}}$$

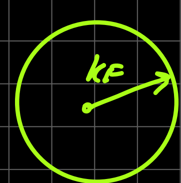
$$\xi_{renom}(k) = Z(k-k_F)\, \partial_k \{\xi_k + Re\,\Sigma(k,0)\}\Big|_{k=k_F}$$

$\downarrow$ assume spherical FS

$$= \frac{k^2 - k_F^2}{2m^*}$$

→ why not k^*_F?

$\Downarrow$



Luttinger theorem
⇒ Volume of FS remains unchanged.

$$\frac{1}{\tau(k,\omega)} = -Z\, Im\, \Sigma(k,\omega)$$

* Let's say $\frac{\partial}{\partial k} Re\left(\Sigma(k,0)\right) = 0$

then $\boxed{\frac{1}{m^*} \sim Z}$

Essence of FL: $Z \neq 0$

destroy FL: $m^* \to \infty$ → phase transitions of a FL (Ref:- Senthil, 2008 paper)

Z → like an order parameter of some kind

$$G(t,k) = \frac{\overset{Z_k}{\underset{\propto\ f^n \text{ of direction}}{}} \; e^{-i\omega t}\, d\omega}{\omega - \xi_k^{renorm} + \frac{i}{\tau(k,\omega)}}$$

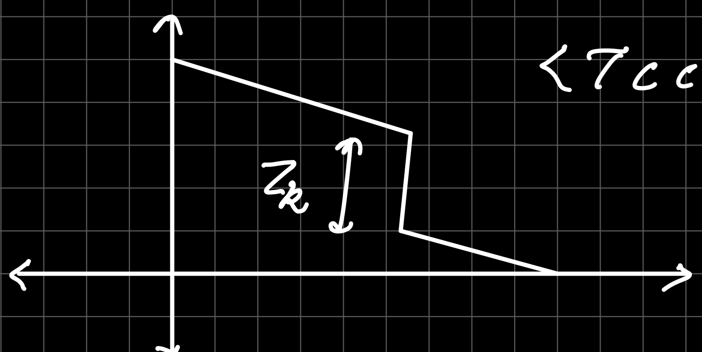
Acc. to Landau argument

$$\tau(k) \sim \frac{1}{(k-k_F)^2}$$ → from phase space arguments

$$G(t,k) = \frac{\overset{Z_k}{\underset{\propto\ f^n \text{ of direction}}{}} \; e^{-i\omega t}\, d\omega}{\omega - \xi_k^{renorm} + \frac{i}{\tau(k,\omega)}} \to (k-k_F)^2$$

$$\xi_k^{renorm} = Z(k-k_F)\,\partial_k \xi \ldots\}$$

$$\therefore G(t, k\approx k_F) \approx Z_k \Big[-i\, \theta(t)\, \Theta(\xi_k^{renorm})\, e^{-it\tilde{\xi}_k} + i\, \theta(-t)\, \theta(-\xi_k^*)\, e^{-it\tilde{\xi}_{k_F}} \Big]$$



$\langle T c c^\dagger \rangle \sim \langle c_k^\dagger c_k \rangle$

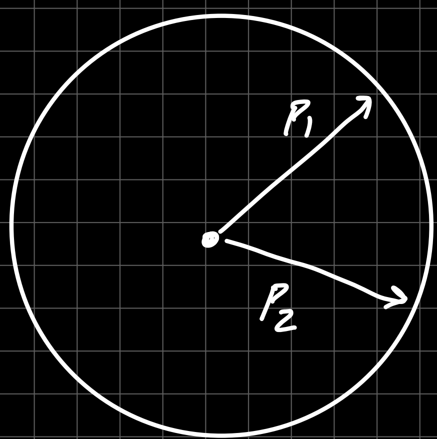
$t = 0^-$

Z_k → can be thought of as

$\underbrace{c^\dagger}_{\text{actual QP creator}} \sim \sqrt{Z}\; \underbrace{f^\dagger}_{\text{bare fermion}} + \underbrace{\cdots}_{\text{stuff we don't know}}$

$Z \to$ overlap b/w bare & QP

$\langle c^\dagger c \rangle \sim \underbrace{Z} \langle f^\dagger f \rangle$



$P_1 - P_2 \longrightarrow P_1', P_2'$

$$\left.\begin{aligned} \mathcal{H}_{Int} &\sim c^\dagger_{k_1} c^\dagger_{k_2} c_{k_1} c_{k_2} \\ &\sim n_{k_1} n_{k_2} \end{aligned}\right\}$$ very loose way of thinking it

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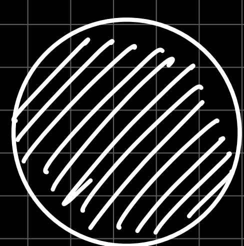
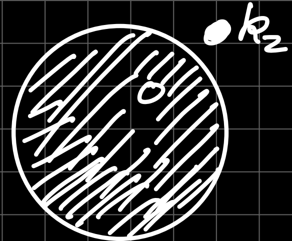
Given distribution of q.p. $n_{p\sigma}$, define

$\delta n_{p\sigma} = n_{p\sigma} - \underbrace{n_{p\sigma}(T=0)}$ = deviation from g.s. occupation

even at finite temp.

= tells us about # of excitations.





$\delta n_k = -1$

$\delta n_k = +1$

[Q. when does a FS break up into smaller puddles?]

Landau $\to$ says that $\delta n_{p\sigma}$ are the actual dof

$\to$ can write down $F(\delta n_{p\sigma})$ & then minimize it.

$$F[\{n_{p\sigma}\}] = E[\{n_{p\sigma}\}] - T\, S[\{n_{p\sigma}\}]$$

$n_{p\sigma} \to 0$ or 1

$\Downarrow$

can use Shannon entropy

Landau postulated a form of E, lacking information.

<u>non.-int.</u>

$$E = \sum_{p\sigma} (\varepsilon^0_{p\sigma} - \mu)\, n_{p\sigma} \qquad n_{p\sigma} = 0,1 \ (@\, T=0)$$

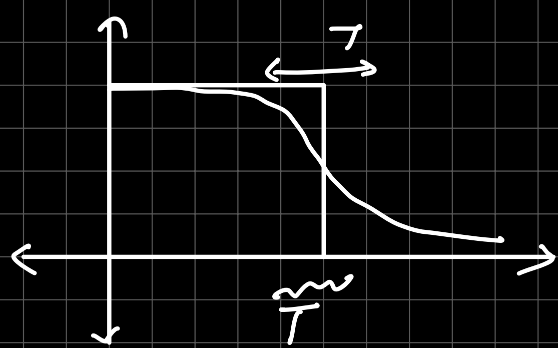
$$\varepsilon^0_{p\sigma} = \frac{p^2}{2m}$$

* Perhaps, then, for interacting case,

$$\epsilon^0_{p\sigma} \to \epsilon_{p\sigma} = \frac{p^2}{2m^*}$$ } but actually this is incorrect.

<u>Why?</u>

our Guess for $E - E(T=0) = \sum_{p\sigma} \underset{\frac{p^2}{2m^*}}{\overset{\approx T}{(\epsilon_{p\sigma} - \mu)}}\ \underset{\frac{T}{\epsilon_F}}{\overset{\approx T}{\delta n_{p\sigma}}}$



$$+ \frac{1}{2} \sum_{\substack{p\sigma \\ p'\sigma'}} \overset{\approx T}{\delta n_{p\sigma}}\ \overset{\approx T}{\delta n_{p'\sigma'}}\ f_{pp'\sigma\sigma'}$$

Landau

we need to keep every term of $O(T^2)$

(coleman has incorrect definition of $\delta n_{p\sigma}$)

(both terms are of the same order)

$$S[\{n_{p\sigma}\}] = -\left[\sum_{p\sigma} (n_{p\sigma} \log n_{p\sigma}) + (1 - n_{p\sigma}) \log (1 - n_{p\sigma})\right]$$

$\therefore$ now get $n_{p\sigma}$

$$\frac{\delta F}{\delta n_{p\sigma}} = 0 \Rightarrow \xi_{p\sigma} + \underbrace{\sum_{p'\sigma'} f_{p\sigma\, p'\sigma'}\, \delta n_{p'\sigma'}}_{} = 0$$

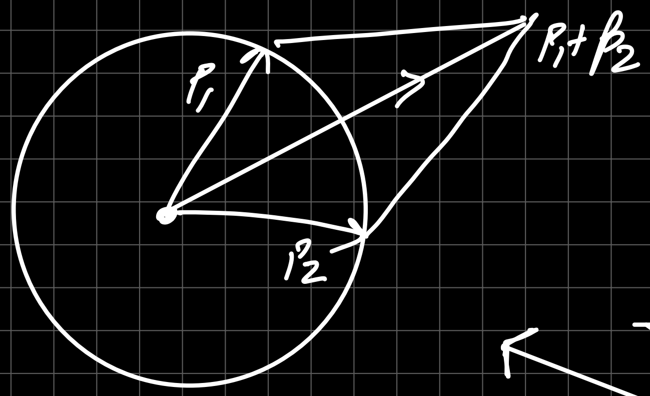
energy of a single particle depends on distribution of all other particles

$$n_{p\sigma} = \frac{1}{e^{\beta(\xi_{p\sigma} - \mu)} + 1}$$

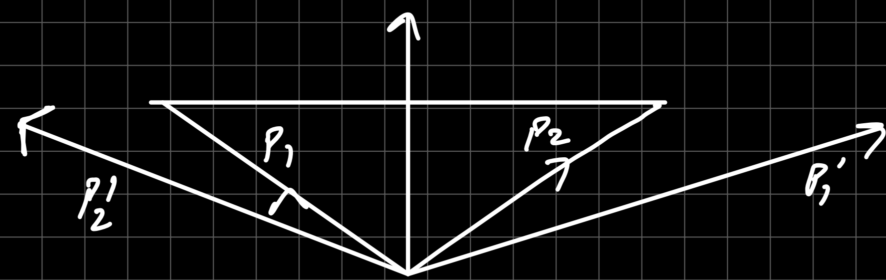
a self consistent equation.

$\xi_{p\sigma}(n_{p\sigma})$

essence of QP $\Rightarrow$ reduce # of parameters

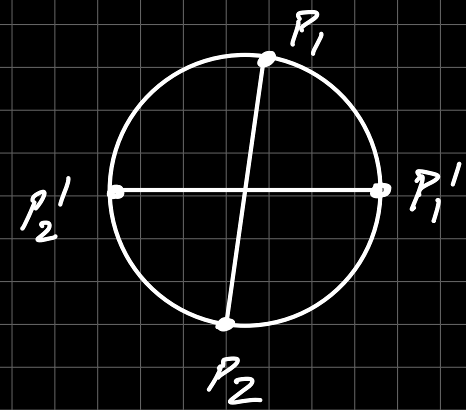


nested FS
↳ some flat portion ↗ in square lattice



$(p_1, p_2) \to (p_1', p_2')$ continuum of solutions } ⇒ ∃ an instability to something

e.g.:- BCS



$p_1' = -p_2'$

$p_1 = -p_2$
↳ large # of solutions
& consequence

$$\chi_{sc}^{(\omega=0)} = \infty$$

$$\langle c^+ c^+ c c\rangle_{fermions} \sim \log(\omega)$$
for $\omega \neq 0$

similarly

$$\int \langle \eta(x,t)\, \eta(0,0)\rangle\, e^{-iqx} e^{i\omega t} \sim \log(\omega)$$

∴ Instability ⇒ continuous family of solution
& due to some RG

nested fermi surface has some instability
↳ log divergences due to multiple solutions